

Chapter 1	Basic Concepts 1 magazino
s: Theory of	Differential Equations 1 Notation 2 Solutions 2 Initial-Value and Boundary-Value Problems 2
Chapter 2	An Introduction to Modeling and Qualitative Methods 9
	Mathematical Models 9 The "Modeling Cycle" 9 Qualitative Methods 10
Chapter 3	Classifications of First-Order Differential Equations 14
ous Differential Wricients 89 81 resqued	Standard Form and Differential Form 14 Linear Equations 14 Bernoulli Equations 14 Homogeneous Equations 15 Separable Equations 15 Exact Equations 15
Chapter 4 ameioffice b	Separable First-Order Differential Equations 21 General Solution 21 Solutions to the Initial-Value Problem 21 Reduction of Homogeneous Equations 22
Chapter 5	Exact First-Order Differential Equations 31
03	Defining Properties 31 Method of Solution 31 Integrating Factors 32
Chapter 6	Linear First-Order Differential Equations 42
	Method of Solution 42

Reduction of Bernoulli Equations 42

8	E.	-	-	A.	10	-	uny
1	N N	d	U	E.	F.		7

Applications of First-Order Differential Equations 50

Growth and Decay Problems 50
Temperature Problems 50
Falling Body Problems 51
Dilution Problems 52
Electrical Circuits 52
Orthogonal Trajectories 53

Chapter 8

Linear Differential Equations: Theory of Solutions 73

Linear Differential Equations 73 Linearly Independent Solutions 74 The Wronskian 74 Nonhomogeneous Equations 74

Chapter 9

Second-Order Linear Homogeneous Differential Equations with Constant Coefficients 83

Introductory Remark 83
The Characteristic Equation 83
The General Solution 84

Chapter 10

nth-Order Linear Homogeneous Differential Equations with Constant Coefficients 89

The Characteristic Equation 89
The General Solution 90

Chapter 11

The Method of Undetermined Coefficients 94

Simple Form of the Method 94 Generalizations 95 Modifications 95 Limitations of the Method 95

Chapter 12

Variation of Parameters 103

The Method 103 Scope of the Method 104

Chapter 13

Initial-Value Problems for Linear Differential Equations 110

CONTENTS xi

Chapter 14 Applications of Second-Order Linear Differential Equations 114

Spring Problems 114 Electrical Circuit Problems 115 Buoyancy Problems 116 Classifying Solutions 117

Chapter 15

131 Matrices

Matrices and Vectors 131 Matrix Addition 131 Scalar and Matrix Multiplication 132 Powers of a Square Matrix 132 Differentiation and Integration of Matrices 132 The Characteristic Equation 133

Chapter 16

eAt

Definition 140 Computation of e^{At} 140

Chapter 17

Reduction of Linear Differential Equations to a System of First-Order Equations

An Example 148 Reduction of an nth Order Equation 149 Reduction of a System 150

Chapter 18

Graphical and Numerical Methods for Solving First-Order Differential Equations

Qualitative Methods 157 Direction Fields 157 Euler's Method 158 Stability 158

Chapter 19

Further Numerical Methods for Solving First-Order Differential Equations

General Remarks 176 Modified Euler's Method 177 Runge-Kutta Method 177 Adams-Bashford-Moulton Method 177 Milne's Method 177 Starting Values 178 Order of a Numerical Method 178

xii CONTENTS

Chapter 20 assert a	Numerical Methods for Solving Second-Order Differential Equations Via Systems 195
	Second-Order Differential Equations 195 Euler's Method 196 Runge-Kutta Method 196 Adams-Bashford-Moulton Method 196
Chapter 21	The Laplace Transform 211
	Definition 211 Properties of Laplace Transforms 211 Functions of Other Independent Variables 212
Chapter 22	Inverse Laplace Transforms 224
	Definition 224 Manipulating Denominators 224 Manipulating Numerators 225
Chapter 23	Convolutions and the Unit Step Function 233
	Convolutions 233 Unit Step Function 233 Translations 234
Chapter 24	Solutions of Linear Differential Equations with Constant Coefficients by Laplace Transforms 242
	Laplace Transforms of Derivatives 242 Solutions of Differential Equations 243
Chapter 25 Tel anoth	
	The Method 249
Chapter 26	Solutions of Linear Differential Equations with Constant Coefficients by Matrix Methods 254
	Solution of the Initial-Value Problem 254 Solution with No Initial Conditions 255
Chapter 27	Power Series Solutions of Linear Differential Equations with Variable Coefficients 262
	Second-Order Equations 262 Analytic Functions and Ordinary Points 262 Solutions Around the Origin of Homogeneous Equations 263

xiii CONTENTS

	Initial-Value Problems 264 Solutions Around Other Points 264
Chapter 28	Series Solutions Near a Regular Singular Point 275
	Regular Singular Points 275 Method of Frobenius 275 General Solution 276
Chapter 29	Some Classical Differential Equations 290
	Classical Differential Equations 290 Polynomial Solutions and Associated Concepts 290
Chapter 30	Gamma and Bessel Functions 295
	Gamma Function 295 Bessel Functions 295 Algebraic Operations on Infinite Series 296
Chapter 31	An Introduction to Partial Differential Equations 304
	Introductory Concepts 304 Solutions and Solution Techniques 305
Chapter 32	Second-Order Boundary-Value Problems 309 Standard Form 309 Solutions 310 Eigenvalue Problems 310 Sturm-Liouville Problems 310 Properties of Sturm-Liouville Problems 310
Chapter 33	Eigenfunction Expansions 318 Piecewise Smooth Functions 318 Fourier Sine Series 319 Fourier Cosine Series 319
Chapter 34	An Introduction to Difference Equations 325 Introduction 325 Classifications 325 Solutions 326
APPENDIX A	Laplace Transforms 330

XiV CONTENTS

APPENDIX B

Some Comments about Technology 336

Introductory Remarks 336 T1-89 337 MATHEMATICA 337

ANSWERS TO SUPPLEMENTARY PROBLEMS 338

INDEX 382